

Constraints on the cosmological parameters from BICEP2, Planck and WMAP

Cheng Cheng and Qing-Guo Huang*

*State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics,
Chinese Academy of Science, Beijing 100190, People's Republic of China*

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In this paper we constrain the cosmological parameters in the base Λ CDM+nrun+r+ n_t model by adopting Background Imaging of Cosmic Extragalactic Polarization (B2), Planck released in 2013 (P13) and Wilkinson Microwaves Anisotropy Probe 9-year Polarization (WP) data, where nrun denotes the running of spectrum index and n_t is the tilt of primordial gravitational waves spectrum. We find that the constraints on r and n_t are $r = 0.23^{+0.05}_{-0.09}$ and $n_t = 0.03^{+0.13}_{-0.11}$ which are compatible with our previous results in arXiv:1403.5463 where only B2 data was used.

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Recently the relic gravitational waves were discovered by Background Imaging of Cosmic Extragalactic Polarization (B2) [1]:

$$r = 0.20^{+0.07}_{-0.05}, \quad (1)$$

and $r = 0$ is disfavored at 7.0σ . Even though a moderately strong tension between B2 and Planck [2] was also reported in [1], the result of B2 is consistent with low- ℓ cosmic microwave background (CMB) data [3], including Planck TT (P13) [2] and Wilkinson Microwaves Anisotropy Probe TE (WP) data [4]. See a similar result for WMAP 7-year data in [5].

After B2 data was released, we constrained the tilt of relic gravitational waves spectrum by using B2 data only, [6], namely

$$r_{0.004} = 0.21^{+0.04}_{-0.10}, \quad n_t = -0.06^{+0.25}_{-0.23}, \quad (2)$$

where the tilt n_t is defined by

$$n_t \equiv \frac{d \ln P_t}{d \ln k}, \quad (3)$$

or equivalently, the amplitude P_t of relic gravitational waves spectrum can be parameterized by

$$P_t(k) = A_t \left(\frac{k}{k_p} \right)^{n_t}, \quad (4)$$

here the pivot scale $k_p = 0.004 \text{ Mpc}^{-1}$ is fixed. Our result provides strong evidence for inflation [7–9], and the alternative models, for example the Ekpyrotic model [10], are ruled out at more than 5σ confidence level. However, our result is different from others [11] and [12, 13] where an apparently blue tilted spectrum of primordial gravitational waves is preferred. Actually our method is different from theirs where they fixed all of the remaining parameters to be those from P13+WP best fit values for Λ CDM+tensor model, or combined B2 with P13+WP in the base Λ CDM+tensor model. We believe that our result in [6] is correct, and a careful explanation and the constraints on the cosmological parameters will be given in the following part of this paper.

It is well-known that the CMB power spectrum generated by primordial gravitational waves are significant only at the low multipoles, e.g. $\ell \lesssim 150$. B2 finds an excess of B-mode power over the base lensed- Λ CDM expectation in the range of $30 \leq \ell \leq 150$ multipoles. Since this range does not cover broad perturbation modes, a power-law spectrum of primordial gravitational waves in Eq. (4) is supposed to be applicable. How the scalar and gravitational waves perturbations affect the CMB TT, TE, EE and BB spectrum is illustrated in the literatures, e.g. [14, 15] etc. Here, for example, see Fig. 1 where the CMB TT spectrum C_ℓ^{TT} is plotted for the tensor-to-scalar ratio fixed to be $r_{0.004} = 0.2$, and the black solid, dashed and dotted curves correspond to $n_t = 0, -2, +2$ respectively. From Fig. 1, we see that a strongly red-tilted spectrum of primordial gravitational waves can make significant contributions to the CMB TT spectrum at very large scales (e.g. $\ell \lesssim 20 \sim 30$) where B-mode power is still absent, and then the CMB TT spectrum can provide a constraint on negative n_t . But the primordial gravitational waves are almost invisible at the small scales, and then the positive n_t can only be constrained by B2 data.

First of all, in order to clarify the discrepancies between our constraint on n_t in [6] and those in [11–13], we need to recall the tension on r between B2 [1] and Planck [2] which prefers a much smaller tensor-to-scalar ratio, namely $r < 0.11$ at 95% C.L.. Actually in [2] Planck group assumed the consistency relation for single-field slow-roll inflation model, i.e. $n_t = -r/8$. Once we relax such a theoretical constraint and take n_t as a fully free parameter, the constraints on r and n_t are quite different from those given by Planck. See the gray contours in Fig. 2. From Fig. 2, we see that the CMB TT spectrum of Planck and TE spectrum of WMAP can significantly constrain the tensor-to-scalar ratio r in the region of $n_t < 0$, but the constraint becomes quite loose in the region of $n_t > 0$. This is consistent with our previous argument. We can also clearly see that both B2 and P13+WP can tightly constrain the relic gravitational waves in the region of $n_t < 0$, but there is almost no overlap between them. However a board region for $n_t > 0$ is still allowed

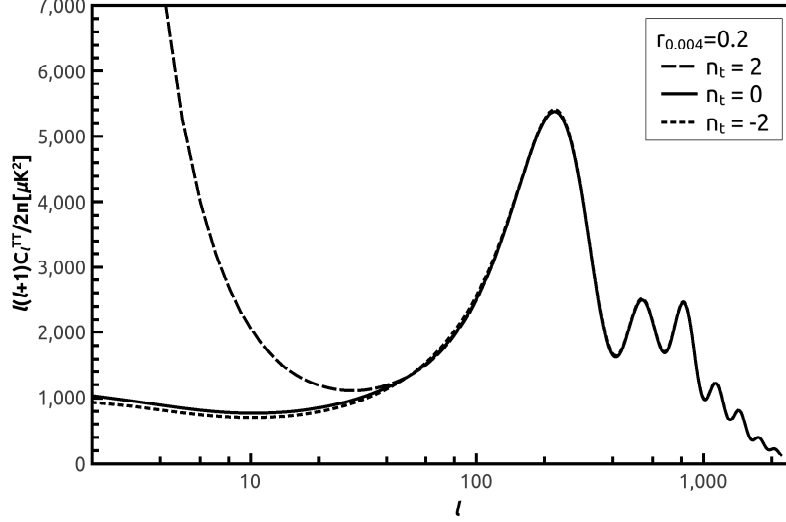


FIG. 1: The plot of C_l^{TT} for different tilt of relic gravitational waves spectrum. Here $r_{0.004} = 0.2$, and the black solid, dashed and dotted curves correspond to $n_t = 0, -2, +2$ respectively.

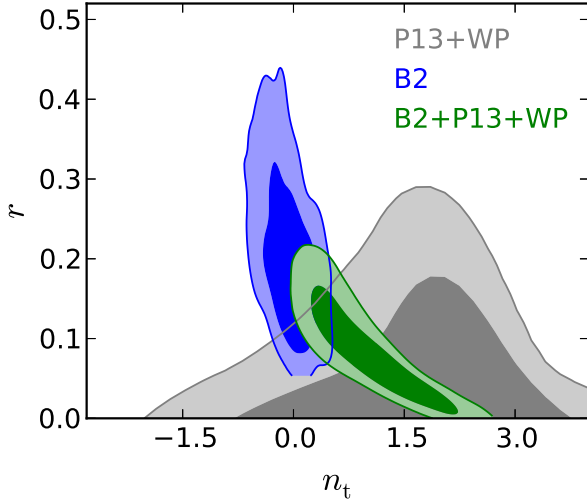


FIG. 2: The contour plot of r and n_t constrained by different CMB datasets in the base Λ CDM+tensor cosmology. The blue, gray and green contours correspond to B2 only in [6], P13+WP and B2+P13+WP respectively.

by P13+WP. That is why the combination of B2 and P13+WP drives the fitting parameter space to the region of $n_t > 0$ (the green contour) in Fig. 2. Technically, from the viewpoint of χ^2 statistics, the tension on the tensor-to-scalar ratio in the region of $n_t < 0$ can significantly increase the χ^2 , and then the fitting parameter space moves to the region of $n_t > 0$. Therefore we conclude that the apparently blue tilted spectrum of primordial gravitational waves in [11–13] might not be reliable

because it can be easily explained by the tension between two CMB datasets, i.e. B2 and P13+WP, which should not be combined together!

In order to confirm our previous argument, we take into account the running of spectrum index which is supposed to relax the tension between P13+WP and B2 [1]. Here we still take the tilt n_t as a free parameter, and the priors for r and n_t are uniform in the ranges of $r \in [0, 2]$ and $n_t \in [-4, 4]$ respectively. We run CosmoMC [16] and work out the constraints on the cosmological parameters from the combination of B2, P13 and WP in the base six-parameter Λ CDM+nrun+r+ n_t model. The results show up in Table I and Fig. 3. From Table I, we see that

Λ CDM+nrun+r+ n_t	B2+P13+WP
parameters	68% limits
$\Omega_b h^2$	$0.02276^{+0.00164}_{-0.00200}$
$\Omega_c h^2$	$0.1161^{+0.0084}_{-0.0079}$
$100\theta_{MC}$	$1.04184^{+0.00103}_{-0.00109}$
τ	$0.1035^{+0.0268}_{-0.0332}$
$\ln(10^{10} A_s)$	$3.106^{+0.005}_{-0.009}$
n_s	$1.031^{+0.029}_{-0.037}$
nrun	$-0.021^{+0.041}_{-0.021}$
r	$0.23^{+0.05}_{-0.09}$
n_t	$0.03^{+0.13}_{-0.11}$

TABLE I: Constraints on the cosmological parameters from the combination of B2, P13 and WP in the Λ CDM+nrun+r+ n_t cosmology.

the constraint on n_t is nicely consistent with our previous one in Eq. (2) [6], and a red-tilted spectrum of relic gravitational waves is consistent with data. One can also

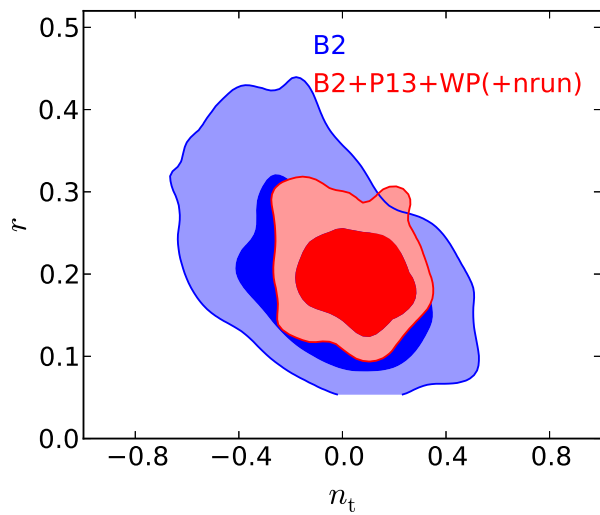


FIG. 3: The constraints on r and n_t in the base Λ CDM+nrn+r+ n_t model from B2, Planck and WP. Here the red contours correspond to the constraints on r and n_t in the base Λ CDM+nrn+r+ n_t model from B2, Planck and WP, and the blue ones correspond to B2 only in the base Λ CDM+r+ n_t model in [6].

consider other possible extensions like those in [11]. We believe that our constraint on n_t does not change so much as long as the tension between B2 and P13 is relaxed in the extensions to the base six-parameter Λ CDM+r+ n_t model.

To summarize, our new constraints on r and n_t from B2, P13 and WP in the base six-parameter Λ CDM+nrn+r+ n_t model are consistent with our previous results in [6]. We believe that our results in [6] and this paper are robust, and we conclude that a red-tilted spectrum of primordial gravitational waves is consistent with current CMB data, and $n_t = 2$ is ruled out at more than 5σ confidence level.

Actually it is dangerous to use the CMB temperature spectrum to constrain the properties of relic gravitational waves, e.g. the tilt n_t , because the relic gravitational waves just make a small contribution to the CMB temperature spectrum which can be affected by a lot of complicated physics, such as the baryon density today, the cold dark matter density today, the spectrum index, the running of spectrum index (nrn), the total mass of active neutrinos ($\sum m_\nu$), the number of relativistic species (N_{eff}), the gravitational lensing, the abundance of light elements and so on. All of these complicated physics can bring strong bias on the data analysis. See, for example, in this paper we constrain the tilt n_t from the same combination of B2, P13 and WP, but the results (see the green contours in Fig. 2 and the red contours in Fig. 3) are quite different from each other in two different models (Λ CDM+r+ n_t and Λ CDM+nrn+r+ n_t). It indicates

that the apparently blue tilted spectrum of relic gravitational waves is not reliable and it can be explained by the tension between B2 and P13+WP in the Λ CDM+r+ n_t model. In a word, the polarization data like those released by B2 is still supposed to be the best one for us to constrain the tilt of primordial gravitational waves spectrum. That is why we only used B2 data to constrain n_t in [6].

Finally, the canonical single-field slow-roll inflation predicts a consistency relation $n_t = -r/8$. We hope that this consistency relation can be explicitly tested in the near future, and then the inflation model will be definitely proved.

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* Electronic address: huangqg@itp.ac.cn

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